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Determination of the apparent transverse and axial dispersion coefficients in a chromatographic column by pulsed field gradient nuclear magnetic resonance

Edgar Baumeister[a], Uwe Klose[b], Klaus Albert[a], Ernst Bayer[a], Georges Guiochon[a,1,*]

[a]Institut für Organische Chemie, University of Tübingen, D-72076 Tübingen, Germany
[b]Abteilung für Neuroradiologie, University of Tübingen, D-72076 Tübingen, Germany

Abstract

Pulsed field gradient nuclear magnetic resonance permits the direct determination of the coefficient of apparent dispersion of a solute in a chromatographic column. Axial and transverse dispersion coefficients can be measured independently as a function of the flow velocity of the mobile phase. Experiments have been carried out using water as the mobile phase, porous C18 silica as the stationary phase, and water as the probe (self-dispersion). The results are reported and compared to similar ones obtained previously by various authors, using conventional chromatographic techniques. The NMR method shows potential for the investigation of the structure and homogeneity of the packing of large diameter columns for preparative chromatography. It permits direct access to local values of these coefficients instead of measuring averages as classical methods do.

1. Introduction

Despite numerous investigations of column performance and of its relationship with the characteristics of the stationary phase, column packing has remained an art to this day. New materials require the development of new packing methods or at least the proper adjustment of conventional methods. The situation is potentially worse in preparative chromatography.

First, conventional packing methods for analytical columns, which use high flow velocities, are unacceptable because the walls of preparative columns cannot be made thick enough to withstand the required high inlet pressures. Second, the column performance and essentially its separation power are critically influenced by large scale fluctuations of the local mobile phase velocity, themselves caused by an inhomogeneous packing density [1]. Deleterious fluctuations of the packing density are more prone to take place in wide columns.

Finally, it is difficult to control the homogeneity of the packed bed when the factors which influence its density and structure are not fully understood, nor even all clearly identified.

* Corresponding author.
[1] Present address: Department of Chemistry, University of Tennessee, Knoxville, TN, 37996-1600 and Division of Chemical and Analytical Sciences, Oak Ridge National Laboratory, Oak Ridge, TN, 37831-6120, USA.

The local value of the permeability is related to the local density of the stationary phase through the external porosity, or fraction of the volume which is not occupied by the particles. Kozeny–Karman equation shows that the permeability increases slightly faster than the fourth power of this porosity [2]. As a result, the local average mobile phase flow velocity may fluctuate significantly across the column cross-sectional area and the surfaces of constant solute concentration are no longer planar and perpendicular to the column axis, as they should be for maximum performance. They become warped. Since the exit section of the column is flat and the whole mobile phase stream exiting from the column is admitted in the same time into the detector cell (in analytical applications) or the fraction collector (in preparative applications), the bands have an apparent width which is enlarged by the contribution of these warps [3].

The structure of a chromatographic bed is complex. The particle size distribution has a finite width and, as a consequence, the formation of a homogeneous, regularly structured bed, which might have been questionable in a cylindrical tube anyway, is impossible to achieve. The distance between particles varies widely across the bed and so does the average diameter of the channels which form between and around the particles. The range of channel diameters extends from 0.1 to 0.5 particle diameter (d_p), with significantly lower or larger values being possible occasionally. These channels are tortuous and constricted [4]. Furthermore, over an average distance of the order of one particle diameter, channels fork and/or merge. Because of the lack of homogeneity of the channels of a packed column at the particle level and of the anastomosis of the various channels available to the mobile phase percolating through the bed, the distribution of linear velocities of the mobile phase across a column section is chaotic, both in space and time. Like in a mountain brook flowing through a layer of boulders, eddies form in the large pools which lie between particles, where the cross-sectional area of a channel increases abruptly. However, by contrast with the quasi-planar surface of the water flow in a

river, these eddies are in a 3-D fluid. Therefore, the eddies do not necessarily have a fixed vertical axis around which to revolve. This axis tumbles and its direction fluctuates randomly. This explains why the flow velocity in any given point of the column may fluctuate chaotically in time.

Accordingly, we are not interested in knowing the exact distribution of the flow velocities in a packed column. The only parameter of importance is the distribution of velocities averaged over scales of space and time which are large compared to a particle but small compared to the distance over which the packing density, permeability, transverse[a] and axial dispersion coefficients and other characteristics of the band broadening and mass transfer kinetics vary significantly. It is obvious that averaging the flow velocity over distances of the order of a few particle diameters will give values which are practically constant in time at a given location and which vary smoothly in all spatial directions. These average values are different from the average flow velocity classically used in chromatography, u, a velocity which is averaged over the entire cross-sectional area of the column and over a period of time which is usually large compared to the band width.

All measurements of the local average velocity across the diameter of a column, be it in chromatography [5–10] or in chemical engineering [11,12], have concluded that significant varia-

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[a] Previous authors [5–8] have considered only the radial dispersion, i.e., the dispersion in the radial direction. This assumes implicitly that the column is cylindrical, which is not strictly correct [8], and that the injection is simultaneous and homogeneous over the entire cross-sectional area of the column inlet. If both assumptions are valid, the radial dispersion coefficient is a function of the distance to the column axis and, possibly, of the column length, but is independent of the direction. If one of these two assumptions is not valid, we must consider the transverse dispersion coefficients, which, at any point of the packing, may vary to a larger or smaller extent, depending upon the direction. Thus, we have to consider the distribution of these coefficients across the whole column. In chromatography, we may assume the column to be nearly cylindrical and consider a radial apparent dispersion coefficient. In NMR, we measure a transverse apparent dispersion coefficient.

tions are observed, with a central core region where the velocity is constant, a peak velocity at a distance from the wall depending on the average size of the packing particles, and a significant drop close to the wall. Such large scale variations of the velocity have a considerable influence on the column efficiency. Knox [13] has suggested that this might be the single most important cause of band broadening in analytical chromatography. This point was never seriously challenged. Admittedly, however, this result was obtained using long, narrow columns, packed with relatively coarse particles ($\rho = d_c / d_p = 10$–40). As a consequence, the wall region where the packing density fluctuates considerably occupies a relatively more important volume fraction of the column than in current short columns, packed with small particles ($\rho = 100$–1000) which have become the standard in HPLC.

Because only transverse dispersion can relax the transverse concentration gradients caused by large-scale drifts of the average velocity in the direction perpendicular to the column axis, it is important to investigate the transverse rate of mass transfer. If the column is homogeneous and if we consider a location which is not very close to the wall, the apparent dispersion coefficient is the same in all transverse directions. For the moderate degrees of angular fluctuations of the packing density which have been observed [8], the variations of the apparent transverse dispersion coefficient with the direction should be small and we can assume the apparent transverse and radial dispersion coefficients to be the same. Because, as a first approximation, non-homogeneous columns can be assumed to have a cylindrical symmetry, it is convenient to characterize the transverse rate of mass transfer by an apparent radial dispersion coefficient or as the radial HETP [5–8] and to compare the rates of axial and radial dispersion. Note also that if the column has a cylindrical symmetry, i.e., if the mobile phase velocity is only a function of r, the distance to the column axis, and not of the angular position, and provided that during the entire injection of the feed the local concentration is independent of the radial position, there is no radial concentration gradient and

only the apparent radial dispersion coefficient matters.

To the extent of our knowledge, the measurement of the radial coefficient of apparent dispersion in liquid chromatography columns has been made so far only by Knox et al. [6] and by Eon [8]. Both groups used rather wide columns (12 and 18 mm I.D., respectively), packed with glass beads, water as the eluent, and p-nitrophenol as the analyte. They measured the axial and radial dispersion of very small samples injected with a syringe whose needle enters into the top layer of the column bed, by recording as a function of time the signal of a polarographic detector. The distance between the microelectrodes of this detector can be adjusted as desired. This work, although quite promising, has not been continued by the systematic investigation of the dependence of the radial dispersion coefficient on the parameters of the column. As a result, a number of issues have been left unresolved. Although not critical for analytical applications of chromatography, these issues have to be revisited in connection with the recent development of preparative liquid chromatography. It has been shown that the column efficiency may have a serious influence on the production rate and the recovery yield achieved with this method [14,15].

Nuclear magnetic resonance (NMR) offers an attractive possibility of monitoring diffusion processes [16] and measuring the dispersion coefficients in the direction of a stream and in the perpendicular direction. The frequency at which a certain nucleus (e.g., a proton) resonates is proportional to the local magnetic field. By superimposing a small magnetic field gradient to the field used to perform NMR measurements (such as is done in a scanner or more elaborately in the 2-D or 3-D Fourier techniques) it is possible to determine the position of a nucleus [16]. This process, called NMR imaging, is frequently used in medicine as a powerful diagnosis tool (NMR tomography). A proper combination of field gradient pulses permits the determination of the change of the position of tagged nuclei after a known period of time. As this signal is a function of the local velocity of

the solvent and because the gradients are applicable in different directions, it is possible to measure separately the axial and transverse components of dispersion. This technique developed after the work of Stejskal and Tanner [17] is called pulsed field gradient spin echo (PGSE) [18] or pulsed field gradient NMR (PFG) [19]. Today, it is frequently used in the determination of diffusion and dispersion coefficients [18,19]. The combination of PGSE and NMR imaging allows space resolved determination of flow, diffusion and dispersion [20,21]. However, depending on the nature of the magnetic field gradient used, different average values of the dispersion coefficient are measured. The smaller the volume over which the averaging takes place, the lower the signal/noise ratio and the more difficult the determination.

The goal of this first publication is a comparison of the results obtained with the conventional methods of measurements of the dispersion in chromatographic columns and by NMR. As we show, this provides a validation of the new approach, and a discussion of the limitations of the results which can be obtained by this method.

2. Theory

In this section, we discuss first the relationships between the axial and radial apparent dispersion coefficients in chromatographic columns and the experimental conditions. In the last part, we review briefly the procedure used for the derivation of these dispersion coefficients from the results of NMR measurements.

2.1. Definitions

The height equivalent to a theoretical plate was defined by Giddings [4] as the slope of the dependence of the band variance on its migration distance. This definition is more general than the classical one, using the square of the ratio of the retention time to the band standard deviation (or using the relationships $N = (L/\sigma_t)^2$, and $H = \sigma_t^2/L$). For example, it allows the

consideration of a plate height function of the distance along the column, which is most useful in GC and TLC. With this definition, the axial and radial reduced HETP's are respectively given by

$$h_a = H_a/d_p = (1/d_p)(\partial\sigma_a^2/\partial z) \tag{1}$$

$$h_r = H_r/d_p = (1/d_p)(\partial\sigma_r^2/\partial z) \tag{2}$$

where H_a and H_r are the axial and radial plate height, respectively, h_a and h_r are the reduced plate heights, σ_a^2 and σ_r^2 are the variance of the band in the axial and radial direction, respectively, and z is the column length. For the purpose of this definition, the band is assumed to have the same variance in all radial directions (cylindrical symmetry). Although this may not be true in actual practice [8,10], deviations from cylindrical symmetry remain small compared to the variation of the dispersion coefficients across the column.

2.2. Plate height equations

As suggested by Knox [22] and confirmed by the results of many experimental investigations [15], the relationship between the axial reduced plate height and the mobile phase velocity is given by

$$h_a = B/\nu + A\nu^n + C\nu \tag{3}$$

where A, B, and C are numerical coefficients, function of the nature of the packing material used, of the analyte studied, and of the homogeneity of the packed bed. n is a numerical coefficient, typically between 0.25 and 0.35 and usually taken equal to 1/3. In their work, however, both Knox et al. [6] and Eon [8] took $n = 0.20$. ν is the reduced velocity or particle Peclet number

$$Pe_p = \nu = \bar{u}d_p/D_m \tag{4}$$

where $\bar{u}$ is the cross-sectional average velocity, d_p the average particle diameter, and D_m the diffusion coefficient in the bulk mobile phase or diffusivity.

In Eq. 3, the structure of the packed bed and eddy diffusion, axial diffusion, and the mass

transfer resistance are characterized by A, B, and C, respectively. A depends on the homogeneity of the packing and is a complex function of the fluctuations of the packing density on short, medium, and long distance. It includes the contribution of eddy diffusion to axial dispersion, and possible contributions of the width of the particle size distribution. Since diffusion is hindered by the presence of the particles, B is smaller than 2 (as would be implied by Eq. 2) and seems, most often, to be close to 1.5. It is a function of the tortuosity of the packing [4]. Because determinations of B require measurements of band profiles at low values of the mobile phase velocity, this coefficient is not known accurately and its possible dependence on the column packing technology has not been studied carefully. C includes the contributions of the kinetics of mass transfer between the stream of mobile phase percolating through the bed and the particles, of the kinetics of mass transfer through the particle pore network, and of the kinetics of adsorption/desorption or more generally of the kinetics of the retention mechanism. The contribution of the kinetics of mass transfer through the mobile phase stream (outside the particles) is included in the coefficient A. In this work, we are mainly interested in the coefficients A and B.

The reduced radial plate height is related to the mobile phase velocity by the Eq. 7

$$h_r = B/v + D \tag{5}$$

where B and v have the same meaning as in Eqs. 3 and D is a coefficient accounting for the contribution of eddy diffusion to radial dispersion. This contribution has been explained in the literature by the phenomenon of "stream splitting", the mechanism of which has been discussed by Horne et al. [7], Saffman [23,24], and Littlewood [25]. When a mobile phase streamlet hits a particle, it separates into several unequal, smaller streamlets which pass between the hit particle and its neighbours, and merge with other streamlets of similar origin. Because the minimum width of these streamlets is very narrow, typically less than $0.1 \, d_p$, they homogenize

quickly. This process accelerates radial dispersion, in much the same way as the entrapment of molecules in the eddies taking place in the large pools between particles enhance axial dispersion.

The plate heights defined in chromatography are related to the corresponding apparent dispersion coefficients, $D_{ap,a}$ and $D_{ap,r}$ by the classical relationships [7]

$$D_{ap,a} = H_a \, \bar{u}/2 = h_a \, v \, D_m/2 \tag{6a}$$

$$D_{ap,r} = H_r \, \bar{u}/2 = h_r \, v \, D_m/2 \tag{6b}$$

These equations permit the conversion of the apparent dispersion coefficients measured by NMR into reduced plate heights or conversely.

2.3. Radial dispersion and stream splitting

This relationship has been discussed in detail by Horne et al. [7], Saffman [23,24], and Littlewood [25]. These authors have demonstrated that a band migrating along a porous bed is dispersed both in the axial and the radial directions by a combination of molecular and convective dispersions. The former results essentially from the axial molecular diffusion, slowed by the fact that migration along a straight line is impossible in a porous bed, as a new packing particle would be hit after a distance which, on the average, is equal to d_p [4,7]. As shown by Saffman [23,24], the results depends on the range of values of the Reynolds and the Peclet numbers encountered in the experiment. The Reynolds number can be written

$$Re = \bar{u} \, d_p \, \rho/\eta = v \, D_m \, \rho/\eta \tag{7}$$

where ρ is the density of the fluid and η its viscosity. Liquid chromatography is typically carried out in a range of reduced velocities from 2 to 100. ρ is of the order of 1 g cm^{-3} and η of the order of $1 \cdot 10^{-2}$ Poise. D_m is between $2 \cdot 10^{-5}$ and $5 \cdot 10^{-7}$ cm^2 s^{-1}. Thus, the Reynolds number is always smaller than 0.2 and is often more than ten times smaller than that. This is the range of *creeping flow*, in which Darcy law is certainly valid [8].

The assumptions made by Saffman are that there is no adsorption of the compound, that the

medium is isotropic and its local average velocity or mean pressure gradient is constant. He considered a model of the column packing consisting of randomly oriented capillary tubes and made a rigorous random-walk treatment. A further approximation is that molecular diffusion is negligible compared to convective dispersion [23]. A more complex derivation, using a more sophisticated model, provided a solution in the case in which molecular diffusion is significant [24]. Since in chromatography the reduced velocity is always larger than 2 and often of the order of 10 or more, we can consider only the results of the first theoretical development

$$D_{ap,r} = \gamma \, D_m + 3 \, \bar{u} \, / \, (16 \, d_p) \tag{8}$$

Hence

$$h_r = 2 \, \gamma / \nu + 0.375 \tag{9}$$

Using a much simpler model, treating the dispersion in an anastomosed network of pores in the same way as molecular diffusion is handled in molecular kinetics, Littlewood [25] derived an equation similar to Eq. 8 for the apparent dispersion and obtained a value of less than 0.40 for the constant term in Eq. 9. His experimental data suggested a value close to 0.20 in gas chromatography.

Horne et al. [7] used the classical random-walk model of band broadening in chromatography developed by Giddings [4], and showed that the constant term in Eq. 9 should rather be between 0.10 and 0.15. Their model assumes the particles to be spherical. When a streamlet impinges on a particle, it divides equally around the sphere, causing the average molecule to undergo a transverse step with an average length of d_p/π [7]. If one lateral step is taken for every $\omega_3 \, d_p$ moved axially (where ω_3 is a number between 0 and 1), the radial variance, σ_r^2, according to the random-walk model is the product of the number of steps $(n = L \, / \, (\omega_3 \, d_p))$ and the square of the step length (d_p/π)

$$\sigma_r^2 = L(d_p/\pi)^2 \, / \, (\omega_3 \, d_p) = L \, d_p/(\omega_3 \, \pi^2) \tag{10}$$

Hence, the contribution, h_c

$$h_c = \sigma_r^2 \, / \, (L \, d_p) = 1 \, / \, (\pi^2 \, \omega_3) \tag{11}$$

It is reasonable to assume that, in a dense packing, one lateral move is made for every progression by one half to one particle diameter, corresponding to ω_3 is between 0.5 and 1. Hence a value of D intermediate between 0.1 and 0.2 [7]. It is obvious, however, that D will vary within this range depending on the packing density. This is probably sufficient to explain the scattering of experimental results found in the literature.

Determination of the apparent dispersion coefficients by NMR

In an homogeneous magnetic field, the spin of a nucleus precesses around the direction of the magnetic field, at the Larmor frequency, ν, given by

$$\nu = \gamma \, B_0 \tag{12}$$

where γ is the gyromagnetic ratio of the nucleus and B_0 is the intensity of the magnetic field. At equilibrium, the total magnetization of the sample is a vector aligned parallel to the direction of the constant magnetic field. In this case, however, the precession of the individual spins remains invisible on a macroscopic scale. All the difficulty in NMR is to produce a non-equilibrium state in which the spin precession is detectable. The application of a 90° RF pulse turns the total magnetization vector into the plane perpendicular to the direction of the main magnetic field. Only this radial component of the magnetization is detectable in NMR. The flexibility of the NMR methods result from the wide variety of the possible approaches to detect this component. The pulsed field gradient spin echo (PGSE) technique [17,18] is a combination of a spin echo sequence (90°–τ–180°–τ–echo), which minimizes the effects of the inhomogeneities of the magnetic field, and of two short field gradient pulses with a time interval Δ for the determination of the dispersion coefficients (Fig. 1). The 180° refocussing RF pulse is applied between the two field gradient pulses.

Since the Larmor frequency is proportional to the intensity of the magnetic field, the superimposition of a magnetic field gradient on a
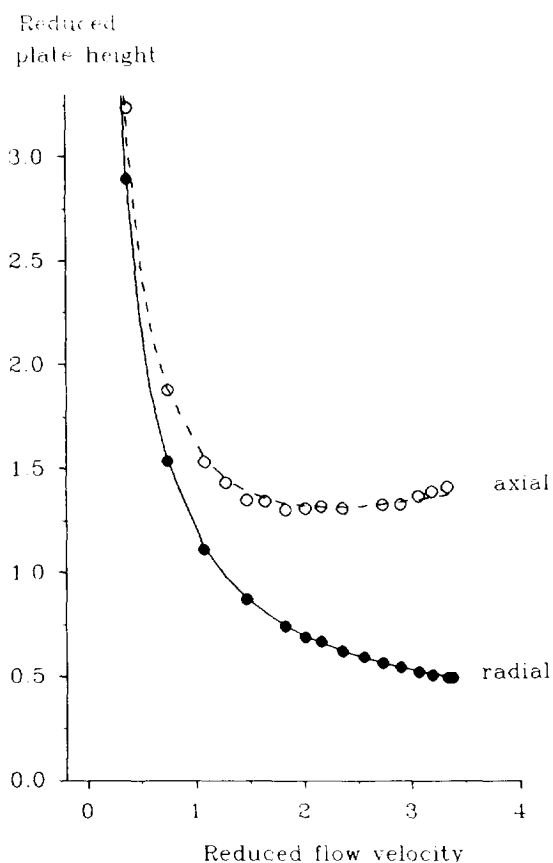
Reduced
plate height

Fig. 1. Reduced axial (○) and radial (●) plate heights as a function of the mobile phase velocity. Column: 15 × 2.6 cm, packed with 15-μm LiChrospher C18 silica. Mobile phase: water. Self-diffusion coefficient of water in the bulk, 2.2 · 10^{-9} m^2 s^{-1}.

constant magnetic field causes protons at different locations to precess at different frequencies. Hence, a short gradient pulse (with a duration, δ, of the order of 10 ms) causes a phase shift which depends on the position of the nucleus. A distribution of the phase shifts within the sample causes a decrease in the signal intensity. The 180° pulse inverts the phase shifts. If the nuclei do not change their positions in the time interval Δ (150 ms), an identical gradient pulse eliminates the phase shift caused by the first gradient pulse. Then, the magnetization would reach the same value as it does when there are no magnetic field gradient pulses. Any movement of the nuclei between the beginning of the first and the end of

the second gradient pulse causes the refocussing to be incorrect. In this case, there is a loss of the amount of magnetization measured. This loss depends on the amplitude of the field gradient, g, and on the dispersion coefficient. It is given by the Stejskal–Tanner equation [17]

$$\ln A/A_0 = -\underline{\gamma}^2 \, D_{ap} \, g^2 \, \delta^2 \, (\Delta - \delta/3) \qquad (13)$$

where A is the signal intensity measured, A_0 is the unattenuated signal intensity (in the absence of field gradient pulses), g is the intensity of the field gradient, and D_{ap} is the apparent dispersion coefficient.

The use of strong field gradient pulses renders negligible the influence of the inherent magnetic gradients. The influence of the relaxation effects is made negligible by keeping constant the pulse duration (δ) and pulse intervals (Δ), while changing the field gradient strength (g). Because the field gradient pulses can be sent in any direction inside the magnet of the NMR instrument, it is possible to measure the apparent dispersion coefficient in any direction, for example in the direction of the mobile phase stream (i.e., axial) or in the perpendicular direction (i.e., radial). Because of limitations due to the signal/noise ratio and the time available for the experiment, the cross-sectional average transverse dispersion coefficient in the only one transverse direction was measured. The column efficiency was high ($h_a \approx 2$) in these experiments, so the apparent transverse and radial dispersion coefficients are equal or at least very close.

3. Experimental

3.1. NMR measurements

The experiments were carried out in the head coil of a 1.5 Tesla whole-body tomograph (Magnetom, Siemens, Erlangen, Germany). We used a spin echo sequence with slice selective excitation RF pulses and a read out gradient at data acquisition. This combination allows the selection of a small slice of the column, perpendicular to its main axis and the acquisition, with the read

out gradient, of the profile of the signal originating from this slice, with spatial resolution in one direction. Thus, the results of the measurements can be related to a well-defined region of the column. Because the gradient pulses induce (magnetic) eddy currents, there was a small space shift in the detected signal. To eliminate this effect, we used only the integrated signal of the profile. Measurements were made at 128 different amplitudes of the field gradient pulses, between 0.2 and 9.7 mT/m. The results were plotted as $\ln (A/A_0)$ against g^2. A straight line was obtained, showing that the influence of the background gradients in our experiments is negligible. The apparent dispersion coefficient is calculated from the slope of this line.

Measurements of the self-dispersion coefficients of water in the chromatographic column measured in the absence of flow gave values of 1.23 and $1.15 \cdot 10^{-9}$ m^2/s for the axial and radial coefficients, respectively. For pure water in an otherwise empty column, the values observed were 2.33 and $2.16 \cdot 10^{-9}$ m^2/s, respectively, while the value generally accepted is $2.4 \cdot 10^{-9}$ m^2/s at room temperature. In still water, the apparent dispersion coefficients should obviously be the same in all directions. Thus, there is an inaccuracy of 5 to 7% in the measurements. It is important to note that, for $\nu = 0$, the ratio D_{ap}/D_m should be equal to $B/2$ (combination of Eqs. 3 and 6a or 5 and 6b). The value obtained is 0.53, in excellent agreement with the value 0.50 derived from the whole set of experimental data (see later).

3.2. Chromatographic system

NMR cannot be carried out on systems incorporating any magnetic metal. Accordingly, the columns used must be made of plastic, glass or silica. We used a silica tube (Merck, Darmstadt, Germany). Accordingly, the inlet pressure is limited to low values, and in spite of the rather large particle size, only values of the reduced velocity below 4 were attainable. The column was packed with 15-μm particles of C18 bonded silica. The column was packed by letting a 30% slurry sediment for three days. This is a marked

deviation from conventional packing procedures. The column exhibited an excellent efficiency. The minimum value of h for toluene and ethylbenzene in acetonitrile–water (80:20) was close to 2). Its packing density was probably lower than usual, which may have affected its long-term stability, but this was never studied. The mobile phase is pure water. A non-magnetic high pressure pump was used. The flow rate was measured manually. The signal recorded corresponds to protons of water, thus giving the self-dispersion coefficient of water. The experiments were conducted at room temperature.

The reduced velocity, ν, is derived from the measured flow rate using the classical relationship $\nu = u \, d_p / D_m = (4 \, F \, d_p) / (\epsilon_T \, \pi \, d_c^2 \, D_m)$, with $d_p = 15$ μm, $D_m = 2.4 \cdot 10^{-9}$ m^2/s, F_v, the measured flow rate, ϵ_T, the total porosity of the stationary phase (0.75), and d_c, the column diameter (26 mm). The reduced plate height is derived from the apparent dispersion coefficient using Eq. 6.

4. Results and discussion

Knox et al. [6] derived the values of the apparent axial and radial dispersion coefficients of p-nitrophenol in 0.1 M aqueous KCl solution, using a 77.5×1.15 cm column packed with 64-μm glass beads. Eon [8] determined the apparent axial and radial dispersion coefficients of the same compound in the same eluent, using 60×1.58 cm columns packed with 76-μm glass beads. In both cases, the axial and radial dispersion coefficients are measured by recording the passage of the zone using a dual polarographic detector 6 with two 0.2 mm diameter microelectrodes. The electrodes can be moved to measure the concentration at any radial position. Injections of very small volumes of sample solution are made with a syringe needle, allowing the introduction into the column of small initial bands and permitting the easy determination of both radial and axial dispersion. Alternately reversing the direction of the flow several times permits a study of the progressive broadening of

the bands in the radial direction [8]. This procedure permitted the determination of the dependence of the axial and radial dispersion on the reduced mobile phase velocity for values of ν between 16 and 250 [6] or 0.6 and 1000 [8]. The axial and radial reduced plate heights were found to be given by the following equations for the data by Knox et al. (KLR) [6] and by Eon [8]

$$h_a(\text{KLR}) = 1.4/\nu + 0.73\ \nu^{0.20} \tag{14a}$$

$$h_r(\text{KLR}) = 1.4/\nu + 0.060 \tag{14b}$$

$$h_a(\text{EON}) = 1.5/\nu + 0.64\ \nu^{0.21} \tag{14c}$$

$$h_r(\text{EON}) = 1.5/\nu + 0.075 \tag{14d}$$

There is an excellent agreement between these two sets of data. As concluded by Knox et al. [6], the axial dispersion coefficient is nearly ten times larger than the radial dispersion coefficient for values of the reduced velocity around 12, a value typical of the conditions used in liquid chromatography, while these coefficients are close for values of the reduced velocities around 1, which are prevalent in thin-layer chromatography. This explains why spots obtained in TLC tend to be circular, while LC bands are more elongated.

Measurements of the apparent self-dispersion coefficients of water were carried out by NMR, on porous silica, using the experimental set-up described earlier in this paper and the procedure whose theory has been explained. The range of

reduced velocities explored is much narrower, between 0.4 and 3.5, hardly one order of magnitude. This is due to the pressure limitations introduced by the use of a silica column. The results are reported in Fig. 2. The apparent axial and transverse (i.e., radial, see previous discussion) dispersion coefficients were found to follow the equations

$$h_a = 0.994/\nu + 0.406\ \nu^{0.2} + 0.204\ \nu \tag{15a}$$

$$h_r = 0.994/\nu + 0.201 \tag{15b}$$

where h_t is the reduced transverse dispersion coefficient. We have adopted the same value of n as found by Knox et al. [6] and Eon [8]. The range of values of reduced velocities explored in our work is too narrow to attempt an accurate determination of the exponent, n, of the eddy diffusion term by introducing an additional degree of freedom when fitting the experimental data to Eq. 3. This limitation results also in part from the need to introduce a mass transfer resistance term, $C\nu$ in Eq. 3 (see below). Accordingly, there are two degrees of freedom in Eq. 3, A and C, the same number as in the problem solved by Knox et al. (A and n). It is legitimate to assume that the first term or axial diffusion term of this equation is the same in Eq. 15b as in Eq. 15a, given the data obtained at low values of the reduced velocity. However, the introduction of another degree of freedom in Eq. 3 would not permit an accurate determination of n. For this purpose, the acquisition of data in a
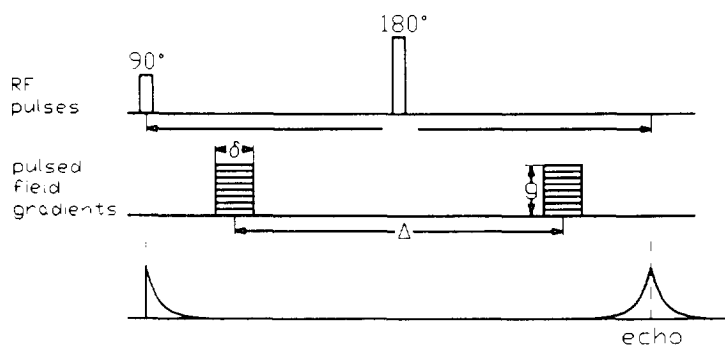

Fig. 2. Events in the PGSE technique. Top, timing of the RF pulses. Middle, timing of the field gradient pulses. Bottom, signal measured.

much wider range of values of v would be necessary.

The three sets of experimental results, those by Knox et al. [6], by Eon [8], and ours, are qualitatively extremely similar. Quantitatively, the values of h_f observed are very close to those reported by these authors [6,8], although the numerical values of the constants B and D are different. The major differences between our results and those previously published are (i) our much smaller value of the tortuosity factor, 0.50 versus a more typical 0.75; (ii) our larger value of the coefficient D, arising from stream splitting; and (iii) the presence of a third term in the equation we used for the axial plate height (Eq. 15a). These three points are discussed in the next paragraphs. We note here, however, that these differences might be explained in a large part (j) by the narrow range of values of the mobile phase velocity explored (with a wider range of v, there might have been better agreement); and (jj) by the difference in the quantity actually measured, the average radial apparent dispersion coefficient, by Knox et al. and Eon, or the average transverse apparent dispersion coefficient, by us. Although the column is highly efficient, some small differences between these two dispersion coefficients are possible.

There is no good explanation for the difference between the values of the B coefficient obtained by Knox et al. [6] and by Eon [8], and the one found in this work. There are few papers in the literature reporting systematic measurements of the coefficient B, as explained above. The general agreement that the tortuosity coefficient, γ, should be close to 0.75 is based on the opinion of Knox [6,7,13,22], itself based mostly on the experimental data reported in his earlier experimental work in gas chromatography [26]. Although there are no reasons to doubt that, in most cases of practical importance in chromatography, B should be of the order of 1.4 to 1.8, it is still unclear whether lower values, of the order of 1, are possible under some particular conditions. Nevertheless, this value is consistent with the ratio of the self-dispersion and self-diffusion coefficients of water measured in still water (flow rate = 0).

The theoretical work of Littlewood [25] concluded that the value of D should be less than 0.40. His experimental results, obtained in gas chromatography, concluded that D was of the order of 0.20. Depending on the columns used, however, values between 0.13 and 0.24 were obtained, the larger values corresponding to the smaller particles. Similarly, Horne et al. [7] concluded that values between 0.10 and 0.20 are more probable than the high theoretical values of 0.375 [23] and 0.40 [25]. These conclusions are in excellent agreement with our experimental results. The fact that our value of D (0.201) is in the high range is probably related to the packing procedure used. Since the chromatographic column used for the NMR determinations was packed by mere sedimentation, its packing density is in the low range of experimentally possible values, suggesting that the largest reasonable value of ω_3 should apply.

Finally, we need to account for the resistance to mass transfer in the particles of packing. Knox et al. [6] and Eon [8] used solid core glass beads with which such a term would be 0 or negligible. In the case of porous silica particles, this is no longer true, even for a small molecule such as water.

5. Conclusion

The great advantage of NMR measurements in this type of studies is that it is possible to determine simultaneously the axial and transverse dispersion coefficients in several locations inside the column. It would be easy also to measure the degree of isotropy of the cross-sectional averaged apparent transverse dispersion coefficient. As a consequence, it is possible to carry out systematic determinations of these local values and to study the dependence of these dispersion coefficients or local plate heights on the position in the column. Systematic determination of local values of h_a and h_r could shed much needed light on the "wall effect" issue. It should also be possible to study the influence of the packing density on the values of

the different parameters of the plate height equation.

Measurements of dispersion coefficients by NMR are done on time and space scales that are orders of magnitude smaller than those on which conventional measurements are made. In chromatography, the space scale is the column length, the time scale is the band width. In NMR, the time scale is Δ. During that time, dispersion causes the molecules to move from their initial position by an average distance l, such that $l^2 = 2 D_{ap} \Delta$. Since $\Delta = 147$ ms and the values of D_{ap} are all larger than $1 \cdot 10^{-9}$ m^2 s^{-1}, and, only in a few cases, at high velocities, are less than four times as large, the average molecule disperses by more than 17 μm and, except in a few experiments, by less than 35 μm. During that time, convection causes the molecules to migrate downstream by between 11 μm ($\nu = 0.5$) and 75 μm ($\nu = 3.5$). Most of these dimensions are only slightly larger than the average size of the pores between the packing particles, which are estimated to be between 0.1 and 0.5 d_p [4–8]. Thus, our results describe the behaviour of the water molecules in a volume equivalent to that of several particles, hence account for the same phenomena as those obtained by conventional techniques. Therefore the two series of data are fully comparable. The use of shorter periods for the field gradient pulses would easily permit a reduction of the volume available to the molecules during the measurement to a small fraction of the average particle volume. If the packing is made of solid core beads, the values obtained for the dispersion coefficients should tend toward those in the bulk liquid when the pulse period decreases. If, however, the particles are porous, a weighted average between the effects of dispersion in the pores of the particles and dispersion in the bulk would be measured. Clearly, the study of the influence of the pulse period could supply new and useful data.

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